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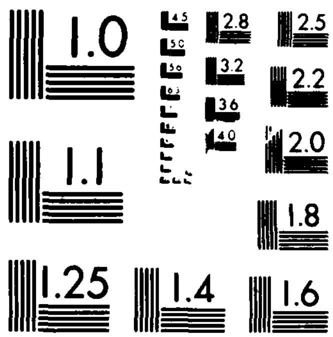

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Research was performed in several means of probabilistic methods, Primary results include representations of equilibrium souburn time distribution in Jackson networks, of queues. Other results develops new practical and theoretical results in computing first passage time statistics. Promising results for developing a method for inverting large sparse motives were obtained.

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1. INTRODUCTION

Ford Aerospace and Communications Corporation (FACC), Western Development Laboratories Division (WDL) is conducting a comprehensive program of research, sponsored in part by the subject contract, on mathematical modeling. The scientific objectives of this research program are to develop improved probabilistic and statistical methods and to demonstrate their power and relevance through application to specific problems in stochastic networks and system reliability.

2. RESEARCH RESULTS ON PROBABILISTIC METHODS

Research efforts to date of the probabilistic methods program at Ford Aerospace are described in a series of seven technical papers. **The fifth, sixth and seventh papers report on research sponsored in part by the subject AFSC Contract.**

1. **"On Failure Modeling,"** A. Lemoine and M. Wenocur.
2. **"A Stochastic Network Formulation for Complex Sequential Processes,"** A. Lemoine.
3. **"On Shot Noise and Reliability Modeling."** A. Lemoine and M. Wenocur.
4. **"Brownian Motion with Quadratic Killing and Some Implications,"** M. Wenocur.
5. **"On Sojourn Time in Jackson Networks of Queues,"** A. Lemoine.
6. **"On First Passage Times and Differential Equations,"** M. Wenocur.
7. **"Algorithms for Inverting Large Sparse Matrices,"** R. Barack and R. Emberson.

The papers listed above are now summarized to provide background and perspective on current problems of interest and further areas for investigation.

2.1 SUMMARY OF RESEARCH REPORTS

The first paper presents a very promising approach to failure modeling which takes account of the dynamic dependency of system failure and decay on the state of the system. Under this approach, system state or wear and tear is modeled by an appropriately chosen random process, eg. a diffusion process; and the occurrences of fatal shocks are modeled by a Poisson process whose rate function is state-dependent. The system is said to fail when either wear and tear accumulates beyond an acceptable or safe level or when a fatal shock occurs. The approach has significant merit. First, it provides new and revealing insights into most of the famous and frequently used lifetime distributions in reliability theory, including the Makeham, Gompertz, Weibull, Rayleigh and Gumbel distributions. In fact, these classic models are obtained in a unified and surprisingly straightforward manner. More significantly, however, the approach suggests intuitively appealing and computationally tractable ways of enhancing these standard failure models and for developing new ones. (*Naval Research Logistics Quarterly, Vol. 32, 1985, pp. 497-508.*)

Stochastic networks, or vector random processes, provide an appealing theoretical framework for modeling complex sequential processes evolving under uncertainty. *The second paper* describes a network formulation for a process wherein an object or "system" moves through a succession of states (nodes) and operating modes (classes) in the course of carrying out its function (fulfilling its purpose). Transitions between states and operating modes occur in a possibly random manner, and require (consume) some resource in randomly varying amounts. The paper discusses the routing behavior and resource requirements of a typical object as it moves through (and eventually out of) the network. We then shift our focus, from a single object and its odyssey, to the network as whole, where time is the resource and many objects are entering the network according to a possibly nonhomogeneous Poisson pattern; in this vein we discuss the evolution of the network over time. Finally, we consider the richness of applications for the formulation and results. (*Naval Research Logistics Quarterly, Vol. 33, 1986, pp. 431-443.*)

The third paper is motivated by the general approach to reliability modeling of the first paper, and explores the implications of shot-noise formulations for system stress. Suppose the system is subjected to shocks or jolts according to a Poisson process with rate λ. Suppose that if a jolt of magnitude D occurs at epoch S then at time $S+t$ the contribution of the jolt to the system stress is $Dh(t)$, where $h(t)$ is a non-negative function, vanishing on the negative half-line. If $\{ S_n, n \geq 1 \}$ are the epochs of shot occurences and $\{ D_n \; n \geq 1 \}$ are the magnitudes of the successive jolts, then the "residual system stress" at time t, say $X(t)$, is given by

$$X(t) = \sum_{n=1}^{\infty} D_n h(t - S_n).$$

Exponential decay of individual shocks corresponds to $h(t) = exp(-at)$. The time to system failure is taken as the epoch of the first count in a doubly stochastic Poisson process with rate function $X(t)$. This formulation is intuitively appealing and gives tractable results. Indeed, a class of failure distributions is derived using the shot noise model for system stress. The properties of these distributions and their structure are explored. (*Operations Research, Vol. 34, 1986, pp. 320-323.*)

The fourth paper carries forward the general approach to reliability modeling in the first paper. System wear and tear is modeled as a Brownian motion with drift. Failure is due to shock only (the system dies in the line of duty). In state x, shocks occur at rate λx^2. Thus, the time to system failure is the "killing time," say τ, of Brownian motion with drift and quadratic killing rate. The paper derives an explicit formula for the state-dependent survival function of this process, that is, $P^x\{ \tau > t \}$. The derivation of this formula is truly stunning, involving the Karhunen-Loeve expansion for Brownian motion, special function theory, and the calculus of residues. Some properties of this killing time distribution are also analyzed. (*Journal of Applied Probability, Vol. 23, 1986.*)

The fifth paper is concerned with representations for equilibrium sojourn time distributions in Jackson networks of queues. For a network with N single-server nodes let h_i be the Laplace transform of the residual system sojourn time for a customer "arriving" to node i, "arrival" meaning external input or internal transfer. The transforms $\{ h_i : i=1,...,N \}$ are shown to satisfy a system of equations we call the *network flow equations*. These equations also lead to a general recursive representation for the higher moments of the sojourn time variables $\{T_i : i=1...N\}$. Combining these formulas with an appropriate martingale, explicit expressions are obtained for the second moment of sojourn time, and the covariance of sojourn time and queue size found upon "arrival," in single-server Markovian queues with feedback; the expression for second moment of sojourn time derived here coincides with the result reported by Takacs in 1963. The paper employs probabilistic methods to obtain probabilistic results. These results and the methods employed hold promise for making further breakthroughs on an important problem which has remained remarkably resistant to analysis. (*Journal of Applied Probability, Vol. 24, 1987.*)

The sixth paper is concerned with practical and theoretical considerations in computing first passage time statistics. The paper is motivated by first passage times as models of failure times. Indeed, this paper continues the line of development initiated in the first paper, where a stochastic process is used to model system state, ie, *wear-and-tear*, and the system fails when either a traumatic killing event occurs (killing events happen with rate $k(x)$ in state x), or the system is retired when *wear-and-tear* reaches some predefined threshold

(ie, a first passage occurs). For example, if system state is modeled as Brownian motion with positive drift, then first passage to a specified threshhold has an inverse Gaussian distribution. This first passage distribution has been successfully applied to numerous problems to obtain good fits. A related but parallel line of development is explored in the fourth paper, where the killing time distribution of Brownian motion with quadratic killing rate is calculated. The aim of this sixth paper is to study first passage time distributions, where the system state process is a general diffusion with reflection at the origin and absorption at $r < \infty$. That is, the system state evolves as a diffusion, and failure occurs at the epoch of first passage (or absorption) to level r . (*Journal of Applied Probability, Vol. 25, 1988.*)

The seventh paper presents a promising method for inverting large sparse matrices of form $I-P$, where I is the identity matrix and P is nonnegative. For P sub-stochastic, matrices of this form are basic to Markovian queueing networks, which represent a fundamental approach to modeling and performance evaluation of large, distributed computer/communications systems, cf. Kelly (1979), and Lemoine (1977, 1986). The algorithm of this paper is quite general, however, and should find application to flow problems in large networks. (In preparation.)

3. DIRECTIONS FOR RESEARCH ON PROBABILISTIC METHODS

Directions for research on probabilistic methods suggested by our results, and which we intend to pursue, include the following:

3.1 PARAMETER ESTIMATION AND DATA ANALYSIS

The state-dependent approach of Lemoine and Wenocur (1985) to modeling system failures presents both significant challenges and opportunities for statistical analysis. In order to apply this approach, one must estimate both state-process parameters and the killing rate function. In most reliability modeling efforts there will be a natural candidate for system state, and the corresponding state process will be observable. The observability permits the study of conditional probability of failure given the state process. Indeed, the conditional distribution of failure due to trauma is simply the distribution of the first event in a nonhomogeneous Poisson process whose rate at time t is equal to $k(X(t))$, where $X(\cdot)$ is the system state process and $k(\cdot)$ is the killing rate function. It is this fact that allows the analyst to decompose the parameter estimation problem into two distinct and loosely coupled statistical estimation problems; namely, the estimation of the killing rate function and the estimation of the system state process parameters.

Furthermore, with the evolution of micro-electronics hardware, there is an ever increasing system self-monitoring capability. The benefits of self-monitoring can only be properly exploited by using state dependent failure modeling. A great deal can be accomplished using data collected by self-monitoring systems It can be used to improve understanding of failure as a function of system state. Such understanding can result in better methods for early failure prediction. An important first step in developing the mathematical techniques needed to exploit monitoring information is embodied by the dynamic failure modeling approach suggested in Lemoine and Wenocur (1985).

We plan to explore estimation problems suggested by the state-dependent approach to modeling system failures, that is, estimation of state-process parameters and killing-rate function. We anticipate that methods of conditional maximum likelihood, cf. Cox and Hinkley (1974), survival analysis, cf. Miller (1981), and multiplicative intensity models for multivariate counting processes , cf. Jacobsen (1982), will be useful.

3.2 COMPUTATIONAL STUDY OF FIRST PASSAGE TIME DISTRIBUTIONS

This line of research will explore computational issues raised in Wenocur (1986). The investigation will compare the usefulness and accuracy of different approaches for computing first passage time distributions. Approaches studied will include a finite spectral expansion, Pearson curve fitting methods, and a direct attack on solving the backward differential equation for the first passage distribution. Of particular interest are the stability, efficiency and accuracy of each these methods.

3.3 SOJOURN TIME IN NETWORKS OF QUEUES

Based on the results of Lemoine (1987), we envision several promising approaches to approximating the higher moments of sojourn times in Jackson networks of queues. In particular we have derived a functional equation which permits us to approximate the second moments. This functional equation has some intriguing properties, eg, it is a positive linear map from $l_1(\pi)$ sequences to $l_1(\pi)$, indeed a contraction mapping possessing a unique solution. We are cautiously optimistic that this approach could lead to a closed form solution for the second moments of sojourn times.

We believe also that martingale methods combined with our representation results for stochastic networks could provide the means to obtain approximations (or exact solutions) to higher moment problems and Laplace transforms.

3.4 QUEUES WITH PERIODIC POISSON INPUT

Queueing systems with non-stationary Poisson input are of definite interest since the customer arrival stream is non-stationary in nature for many service systems. The fundamental issue is to what extent variations in the input process increase system load and congestion. Some recent progress has been reported by Rolski (1981, 1986), Lemoine (1981), and Heyman (1982). But there are several basic and important problem areas in need of further study and development.

The goal of our research effort on queues with periodic Poisson input is to obtain approximation and comparison results for the family of ergodic distributions identified in Harrison and Lemoine (1977). The point of departure for our investigation is a basic characterization result developed in Lemoine (1981).

4. RELATED CONTRACT ACTIVITY

Supported in part by the subject AFSC contract, A. Lemoine and M. Wenocur attended the Special Interest Conference, "Queueing Networks and Their Applications." This conference was held at the Hyatt Regency Hotel in New Brunswick, New Jersey, January 7-9, 1987.

The meeting was sponsored by the Operations Research Society of America (ORSA), The Institute of Management Sciences (TIMS), and The Society for Industrial and Applied Mathematics (SIAM). Support was provided by AT&T Bell Laboratories, AT&T Technologies, the Department of Industrial Engineering at Rugters University, and the National Science Foundation.

Networks of queues have been the focus of much attention in recent years. This has been motivated by both a wealth of interesting and important applications and a stimulating and fruitful area for research. The purpose of this conference was to provide a forum for researchers and practitioners to present their current work and discuss new problems.

The conference was organized around five main topics:

- Theory
- Approximations
- Manufacturing
- Computers and Communications
- Simulation

The conference included also a demonstration session for analytical and simulation software packages.

This conference was well organized and attended, with over two hundred (200) participants from the United States, Canada and Western Europe. It was a highly interesting and very informative meeting.

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